Analysis of the bicycle change strategy for hilly time-trials

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Abstract

The uphill time-trial is a cycling race in which there is at least one mountain pass or a combination of low and high gradient sections. Usually, climbing cyclists achieve good results in uphill time-trials; however, in addition to the cyclist’s ability, the race strategy is also important for achieving good results. As part of the race strategy, the selection of the type of bicycle is important. The time-trial bicycles usually reduce aerodynamic drag while the traditional road bicycles are lighter. Taking into account the road gradient profile of the race, as part of the bicycle selection strategy, the cyclist can change the type of bicycle during the race to take advantage of each one in specific sections of the route. This paper presents a methodology for planning the bicycle change strategy for some ideal routes with simplified road gradient profiles. An optimization problem is stated to minimize the race time and to find the location on the route where the bicycle change must be done. The methodology is applied to three simplified road profiles to define if the bicycle change strategy is beneficial when different cyclist’ power output levels are analyzed.

Keywords: Bicycle, cycling, longitudinal dynamics, race time optimization, uphill time-trial.

Resumen

La cronoescalada es un tipo de competencia de contrarreloj en bicicleta que se caracteriza por tener un ascenso de montaña o una mezcla de secciones planas y ascensos. En este tipo de competencia se suelen destacar los ciclistas con gran habilidad de ascenso. Sin embargo, adicional a la habilidad del deportista, la estrategia de carrera también es decisiva. Parte de la estrategia incluye la selección adecuada de la bicicleta: en general las bicicletas de contrarreloj tienen ventaja aerodinámica mientras que las bicicletas de ruta tradicionales son más livianas. Dependiendo del recorrido a realizar, una alternativa es realizar un cambio de bicicleta para aprovechar la ventaja que cada una de ellas ofrece. El objetivo del presente trabajo es planear la estrategia de cambio de bicicleta en carreras de cronoescalada. Se plantea un problema de optimización que busca minimizar el tiempo de carrera y determinar el punto de la ruta en el cual debe realizarse el cambio de bicicleta. Para tres rutas se define si la estrategia de cambio de bicicleta disminuye el tiempo de carrera para diferentes niveles de potencia.

Palabras clave: Bicicleta, ciclismo, contrarreloj, dinámica longitudinal, optimización, tiempo de carrera.
1. Introduction

In individual time-trial races, each cyclist competes alone and has to keep the highest speed as possible throughout the course to achieve the minimum race time. One type of individual time-trial is the hilly time-trial, during this race the individual performance is evaluated in one or more mountain climbs. For this type of competition, riders have to plan a different racing strategy than the one used in a time-trial with a low road gradient. Several studies have been focused on the best pacing strategy for time-trial (1-3) and hilly time-trial races (4). Other studies have been focused on determining an optimal cyclist position that minimizes aerodynamic drag and maximizes the cyclist’s physiological performance (5,6). Likewise, some studies have performed laboratory tests to determine the cyclist’s performance parameters that can be used to predict performance during the time-trial and hilly time-trial races (7,8). In addition to the pacing strategy, the selection of cycling equipment is also important for the race strategy. Some works have been focused on the evaluation of the performance of helmets, clothes, and wheels (9-11). However, for the best knowledge of the authors, a methodology to analyze the selection of the bicycle to be used, especially during the uphill time-trial, has not been reported in the literature.

During the last editions of the three cycling grand tours, several individual time-trial stages have been hilly time-trials. For example, in the Tour de France, stage 17 in 2013 was characterized by two climbs of around 300 meters and a downhill section at the end; stage 18 in 2016 was characterized by an initial section with low slopes of around 4 km followed by a climb of around 400 meters. In the Vuelta a España, stage 11 in 2012 and stage 11 in 2014 were characterized by a combination of flat sections, an ascent of around 500 meters and a final descent. In the Giro d’Italia, several individual time-trial stages had mountain climbs: stage 16 in 2010 with a climb greater than 1000 meters; stage 16 in 2011 with a flat section followed by a climb of around 600 meters; stage 18 in 2013 with an ascent of more than 1000 meters and a high variability in the gradient; stage 19 in 2014 with a first flat section of around 7.5 kilometers followed by a climb of 19 kilometers with an ascent close to 1500 meters; and stage 15 in 2016 with an ascent close to 800 meters. Additionally, during the road world championships in 2017, in the elite men’s time-trial race, the route finished with a climb of 300 meters of 3 kilometers length after an initial section of around 28 kilometers with a low road gradient. Taking into account this type of hilly time-trial routes, the selection of the type of bicycle is an important part of the race strategy. In general, a time-trial bicycle is better in low gradient sections where the highest resistive force is the drag force. Likewise, a road bicycle, usually lighter than the time-trial bicycle, is better during the climbs in which the largest resistive force is the component of the weight due to the gravitational force. In some of the aforementioned race stages in grand tours, as well as in the road world championships in 2017, some cyclists decided to switch their bicycles at some point along the route. The change seeks to take advantage of each bicycle in a particular section of the route. Nonetheless, the decision of whether to make the switch and its location is not trivial; proof of this is the different choices within the cyclists. For example, in stage 19 of the Giro d’Italia 2014, many cyclists decided not to switch the bicycle even though within the top 8 of the stage seven cyclists made the switch. As another example, in road world championships in 2017, two cyclists in top 3 decided not to make the switch, and according to the UCI, 41.5% of the total cyclists decided to switch their bicycles.

This paper aims to develop a methodology to analyze the bicycle change strategy for hilly time-trial races. The analysis is based on the simulation of the longitudinal dynamics of the bicycle-cyclist set for different routes using two types of bicycles and different power levels delivered by the cyclist. An optimization problem is stated for minimizing the race time by defining the optimal bicycle change point. The results allow one to determine for each route if it is convenient to make a bicycle change (including the time it takes to the cyclist to change the bicycle) and if so, determine the point along the route at which the change should be made.
2. Methodology

The methodology seeks to evaluate the longitudinal dynamics of two types of bicycles for specific hilly time-trial routes. Based on a simplified elevation profile of the route, the race speed and time are obtained as a function of distance for a given power delivered by the cyclist. Based on this information, an optimization model is proposed to minimize the race time based on the distance at which a change should be made between the time-trial and the road bicycle.

2.1 Determination of a simplified elevation profile

For the present work, simplified routes based on real routes of the highlands areas in the “altiplano Cundiboyacense” zone in Colombia are used. The altitude as a function of the distance of the real route is fitted with a quadratic function. In this manner, an approximation of the elevation profile is obtained as a continuous and derivable function that allows one to determine the gradient of the route as a function of the distance. Eq. (1) shows the function that approximates the elevation profile of the route and Eq. (2) shows the function that approximates the slope.

\[ h(x) = p_1 x^2 + p_2 x + p_3 \, [m] \]  
\[ s(x) = \frac{dh(x)}{dx} = 2p_1x + p_2 \, \left[ \frac{m}{m} \right] \]  

\( x \) is the distance, \( h(x) \) is the altitude, \( s(x) \) is the slope, and \( p_1, p_2 \) and \( p_3 \) are the coefficients of the fitted function that describes the elevation and slope of the simplified route.

2.2 Dynamic model

For the simulation of the dynamics of the bicycle-cyclist set, a longitudinal dynamics model is used. Eq. (3) describes the model based on the acting forces.

\[ M \frac{d^2x}{dt^2} = F_x - R_x - D_x - G_x \]  

\( M \) is the mass of the bicycle-cyclist set, \( t \) is the time, \( F_x \) is the traction force in the rear wheel, \( R_x \) is the rolling resistance (Eq. (4)), \( D_x \) is the aerodynamic drag force (Eq. (5)), and \( G_x \) is the force component of the weight due to gravity (Eq. (6)).

\[ R_x = Mg f_r \cos(\theta) \]  

In Eq. (4), \( g \) is the gravitation constant and \( f_r \) is the rolling resistance coefficient.

\[ D_x = \frac{1}{2} \rho C_d A \left( \frac{dx}{dt} \right)^2 \]  

In Eq. (5), \( \rho \) is the air density, \( C_d \) and \( A \) are the drag coefficient and frontal area respectively.

\[ G_x = Mg \sin(\theta) \]  

In Eq. (6), \( \theta \) is the slope angle of the route, positive in ascents. Eq. (7) shows the slope angle based on the elevation function in Eq. (2).

\[ \theta = \tan^{-1}[s(x)] \]  

By multiplying the Eq. (3) by the speed, the model can be rewritten as an energetic expression of the power delivered by the cyclist as in Eq. (8), where \( \eta \) is the powertrain transmission efficiency.

\[ \eta P = M \frac{d^2x}{dt^2} \frac{dx}{dt} + Mg f_r \cos(\theta) \frac{dx}{dt} + \frac{1}{2} \rho C_d A \left( \frac{dx}{dt} \right)^3 + Mg \sin(\theta) \frac{dx}{dt} \]  

For a given route with a known slope and a given power delivered by the cyclist, the differential equation in Eq. (8) can be numerically solved for determining the race speed and time.

Both the product of the drag coefficient and the frontal area \( C_d A \) and the bicycle-cyclist set mass \( M \) are defined for each bicycle. Those parameters represent the performance of each bicycle in the model. Table 1 shows the values of the parameters used for each bicycle.
Table 1. Parameters in the dynamic model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Time-trial bicycle</th>
<th>Road bicycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CdA \text{ [m}^2\text{]}$</td>
<td>0.26</td>
<td>0.33</td>
</tr>
<tr>
<td>$M \text{ [kg]}$</td>
<td>89</td>
<td>87</td>
</tr>
<tr>
<td>$f_r \text{ [-]}$</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>$\eta \text{ [-]}$</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>$\rho \text{ [kg/m}^3\text{]}$</td>
<td>0.90</td>
<td>0.90</td>
</tr>
</tbody>
</table>

$CdA$ is based on the values of $Cd$ and $A$ reported in literature (5, 9, 12, 13). $f_r$ is an average value of the rolling resistance coefficient for a 700x23 tire inflated at 8 bar (14). In Kyle (15), typical values of the transmission efficiency at different power levels are reported. The air density value is calculated based on the model by Picard et al. (16) using the typical weather conditions in the area.

Using Eq. (8), it is possible to determine the race time as a function of the distance $t_1(x)$ that takes to the time-trial bicycle from the start to the $x$ position along the route, as well as the race time $t_2(x)$ that takes to the road bicycle from the start to the $x$ position.

2.3 Optimization problem

The bicycle change strategy is based on taking advantage of the lower drag force of the time-trial bicycle (more aerodynamic components and cyclist’s posture) in the low gradient sections and the lower mass of the road bike in climbs. In this work, routes that start with a low gradient and gradually increase are analyzed. Therefore, the problem is restricted to start the race with a time-trial bicycle and make a single switch to a road bicycle. The optimization problem focuses on determining the distance at which a single bicycle switch must be made to minimize the race time.

The variable $x_c$ is defined as the distance where the bicycle change occurs, and $L$ represents the total distance of the route; therefore, the total race time $t_t$ can be defined as a function of the bicycle change distance as in the Eq. (9).

$$t_t(x_c) = t_1(x_c) + t_2(L) - t_2(x_c)$$  \hspace{1cm} (9)

The optimization problem for determining the point along the route in which the bicycle change minimizes the race time is defined in Eq. (10).

$$\min t_t(x_c)$$  \hspace{1cm} [x_c]  \hspace{1cm} (10)

subjected to

$$0 \leq x_c \leq L$$

The indexes are defined as in Eq. (11) and Eq. (12) respectively. Those indexes are used to quantify the strategy advantage. Compares the race time when only using a time-trial bicycle in regard to using the bicycle change strategy. Conversely, compares the race time when only using a road bicycle in regard to the bicycle change strategy. $x_c^*$ is the optimal distance of bicycle change.

$$t_{v_1} = t_1(L) - t_t(x_c^*)$$  \hspace{1cm} (11)

$$t_{v_2} = t_2(L) - t_t(x_c^*)$$  \hspace{1cm} (12)

The variable $t_{vf}$ in Eq. (13) compares the race time when using the best one-bicycle-choice (depending on the power delivered by the cyclist) in regard to the race time when using the bicycle change strategy.

$$t_{vf} = \min[t_1(L), t_2(L)] - t_t(x_c^*)$$  \hspace{1cm} (13)

The variable $t_{vt}$ (Eq. (14)) determines the real advantage when including the time that takes to the cyclist switch between bicycles $T_k$.

$$t_{vt} = \begin{cases} 
0, & \text{if } t_{vf} \leq T_k \\
t_{vf} - T_k, & \text{if } t_{vf} > T_k 
\end{cases}$$  \hspace{1cm} (14)

For $t_{vt} > 0$, it is beneficial to change the bicycle at $x_c^*$; for $t_{vt} = 0$, changing the bicycle is not beneficial, and the cyclist should race only in the time-trial bicycle.
2.4 Bicycle change time

An on-road experiment was performed to measure the time that takes to the cyclist change from one type of bicycle to the other. In a circuit of approximately 1500 meters in length, a cyclist made two tests. In the first test, the cyclist rode ten laps at a nominal speed of 30 km/h and the total time \( T_{ex,1} \) was recorded. In the second test, a point in the circuit was established in which the cyclist had to completely stop, get off the bicycle and place both feet on the ground, get back on the bicycle and reach again the nominal speed of 30 km/h; the cyclist rode ten laps following the aforementioned process for each lap and the total time \( T_{ex,2} \) was recorded. Based on Eq. (16), the average bicycle change time \( T_k \) was obtained for the conditions established in the experimental test. This time is assumed constant in this work; it represents a particular case according to the experiment carried out.

\[
T_k = \frac{T_{ex,2} - T_{ex,1}}{10} = \frac{1852 - 1749}{10} = 10.3 \text{ s}
\]  

(16)

3. Results and discussion

Based on the presented methodology, three simplified routes with different gradients and lengths were analyzed. For each case, the results of the optimization problem for different power levels are presented.

3.1 Study cases: simplified routes

Three routes were analyzed, the elevation profiles were fitted, and the slope functions were determined. The distance of each route was approximated to 10, 20 and 30 km respectively. The first route starts with a slope about zero, and it quickly increases up to 7%. The second route starts with a low slope that slowly increases without exceeding 4%. The third route starts with a descent of about 6 km length; then the slope gradually increases up to about 12%. Table 2 shows the coefficients of the second order polynomial for each altitude profile according to Eq. (1).

<table>
<thead>
<tr>
<th>Route</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>( L [km] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.5x10^{-7}</td>
<td>-1.2x10^{-3}</td>
<td>2631.7</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>8.0x10^{-8}</td>
<td>4.6x10^{-4}</td>
<td>2482.1</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>2.5x10^{-7}</td>
<td>-3.1x10^{-3}</td>
<td>2480.0</td>
<td>30</td>
</tr>
</tbody>
</table>

Figure 1 shows the simplified elevation profile and the slope (Eq. (2)) of the three routes.

3.2 Race speed and time

Based on the longitudinal dynamic model, the race speed is calculated as a function of distance. The model is evaluated with different power levels delivered by the cyclist to take into account different levels of training. It is assumed that the cyclist delivers a constant power throughout the course. The power range evaluated goes from 100 W up to 300 W. Figure 2 shows the race speed for each of the routes with two power levels. For both power levels, it can be seen that at the start of each route, where the slope is low, the time-trial bicycle allows the cyclist to maintain a higher speed. For the high power level, the difference in speed between both bicycles at the beginning of
the route is larger, and as the slope increases, the speed difference becomes smaller. For both power levels on route 1 and 3, the road bicycle allows the cyclist to maintain a higher speed at the end of the route; for route 2, the increase in the power level allows the cyclist to maintain a higher speed with the time-trial bicycle along the entire route.

Figure 2. Race speed as a function of the route distance. (a) Route 1, (b) route 2, (c) route 3.

Table 3 shows the total race time if only one type of bicycle is used. The results for power levels of 100 W and 300 W are presented. For the three routes analyzed, if only one bicycle is used, with a power of 300 W, the time-trial bicycle is better. For a power of 100 W, the best bicycle varies according to the route.

Table 3. Race time for each bicycle under two power levels: 100 W and 300 W

<table>
<thead>
<tr>
<th>Route</th>
<th>100 W</th>
<th>300 W</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Race time with time-trial bicycle [min]</td>
<td>Race time with road bicycle [min]</td>
</tr>
<tr>
<td>1</td>
<td>61.6</td>
<td>61.0</td>
</tr>
<tr>
<td>2</td>
<td>85.4</td>
<td>85.5</td>
</tr>
<tr>
<td>3</td>
<td>239.9</td>
<td>236.8</td>
</tr>
</tbody>
</table>

3.3 Optimal distance of change

The behavior of the objective function of the optimization problem for two power levels for each route is shown in Figure 3. For route one, at a power level of 100 W, the optimum distance of change is 3.2 km, while for a power of 300 W it is 7.1 km. For route two, for a power of 100 W, the optimum distance of change is 10.0 km, and at 300 W it is 20 km. For route three, for a power of 100 W the optimum distance is 10.6 km, and for 300 W it is 16.2 km.

It should be noted that for route two at a power of 300 W, the bicycle change distance is equal to the total distance of the route. This result indicates that the change strategy for this route and power level is not beneficial and the cyclist should race only in the time-trial bicycle.

Figure 3. Race time as a function of the distance of change. (a) Route 1, (b) route 2, (c) route 3.

Figure 4 shows the optimal distance of change for each of the routes analyzed under different power levels. The optimal distance is shown normalized with respect to the total distance of each route. It can be seen that the optimal distance of change increases as the power level increases. For route
two, from 208 W, the race strategy is not to change the bicycle and race only on the time-trial bicycle.

Figure 4. Ratio between the optimal distance of change and route length as a function of the power delivered by the cyclist.

Figure 5 shows the indexes $t_{v1}$ and $t_{v2}$ for each route. Figure 6 shows $t_{vf}$ as a function of the power for each route. For the route two, starting from 208 W, $t_{vf} = 0$ given that there is no advantage when doing the bicycle change as shown before.

Figure 7 shows the time advantage $t_{vt}$ for each of the routes. For route one, the bicycle change strategy is beneficial up to 230 W and the largest advantage is at 166 W corresponding to 11.8 seconds. For route two, the bicycle change is only useful up to 134 W; the largest advantage is at 100 W corresponding to 21.6 seconds. For route three, the bicycle change strategy is useful throughout the range power analyzed; the largest advantage is obtained at 100 W corresponding to 69.7 seconds.

3.4 Discussion

The dynamic model is used to determine the speed as a function of distance based on a constant power delivered by the cyclist. Based on specific parameters
for each bicycle-cyclist set, the performance in each bicycle is evaluated. The power level as an input to the dynamic model allows one to analyze different training levels of the cyclist.

The proposed optimization model is used to determine the bicycle change distance that minimizes the race time. The proposed $t_{bc}$ index compares the time advantage when following a bicycle change strategy with respect to using only one type of bicycle.

It is essential to include the bicycle change time. For the analysis of routes shown in this work, a particular experiment was proposed to determine the bicycle change time. In the experiment, the power for the acceleration phase was not restricted, and additionally, the acceleration phase was performed in a circuit with a slope close to zero. For the stop and acceleration phases at different nominal speeds or starting from higher slopes, the bicycle change time might vary. Moreover, depending on the cyclist’s ability, this time of change also might change.

The proposed methodology determines if a bicycle change should be made and what is the optimum distance of change. In the routes evaluated in the study cases, it is evident that the bicycle change is beneficial at low power, and moreover, that the advantage times obtained can be decisive in competition.

The three routes used as cases of study are of particular interest to the authors; the aim is to present the results of the methodology when evaluating scenarios with particular slopes and lengths. The methodology can be applied to real routes in which the slope information is available, or the altitude information can be suitably processed. It is worth highlighting that the results can be highly sensitive to the slope information of the route.

For the three routes analyzed, the power levels at which the bicycle change strategy decreases the race time were determined. Each route has different distances and gradients, so the results cannot be generalized. Each route must be studied independently to analyze the behavior of the race speed depending on the characteristics of each cyclist and types of bicycles. Due to the number of factors to be evaluated and the particularities of the bicycle-cyclist sets, it is not trivial to determine whether to change the bicycle or not and therefore an analysis methodology is required.

4. Conclusions

A methodology was proposed for planning a change strategy for hilly time-trial races when using two types of bicycles. Based on an elevation profile, a longitudinal dynamics model and an optimization model were used for finding the optimal bicycle change distance that minimizes the race time. Some indexes were proposed for determining the advantage of the bicycle change strategy regard to competing only in one type of bicycle. The result of the methodology defines whether the bicycle should be changed and at what point of the route; the results are presented for different power levels delivered by the cyclist.

Three simplified elevation profiles were used as study cases; each route has particular characteristics of distance and slope. For the first two routes, it was determined that only for low power levels there is a benefit by making a bicycle change, while for the third route a benefit is obtained by making a bicycle change in the whole power range analyzed.

5. References


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